WORCESTER POLYTECHNIC INSTITUTE
THIRTIETH INVITATIONAL MATH MEET
OCTOBER 17, 2017
INDIVIDUAL EXAM QUESTION SHEET WITH ANSWERS

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes long. Questions 1-4 are each worth 1 point. Questions 5-8 are each worth 2 points. Questions 9-11 are each worth 3 points. Calculators and other electronics **MAY NOT** be used.

1. Simplify $\sqrt{-9} + \sqrt{-16}$

   Ans: 7i

2. Factor $2x^3 - x^2 - 8x - 5$

   Ans: $(2x-5)(x+1)^2$

3. Convert the base 10 number $29\dfrac{13}{64}$ to binary

   Ans: 11101.001101_2

4. What is $\cos(179\pi/6)$?

   Ans: $\sqrt{3}/2$

5. If $f(x) = (3x-2)/(x+1)$ what is the inverse function $f^{-1}(x)$?

   Ans: $(2+x)/(3-x)$

6. Find x such that $\sqrt{x+23} + \sqrt{x} = 23$

   Ans: x=121

7. What is the units digit of the number
   $1 + 9 + 9^2 + 9^3 + \ldots + 9^{1988} + 9^{1989}$?

   Ans: 0

8. Solve for x: $(\log_{10} x)^3 = \log_{10} x^4$

   Ans: 100, 1/100, 1

9. 5 years ago, the ratio of a father's and his son's ages was 3:1.

   15 years from now, the ratio will be 2:1.

   What is the father's current age?   Ans: 65

10. Find any x which satisfies

$$\left(\frac{1}{25}\right)^x (125)^{x^2} = (125)^x (\frac{1}{25})$$ Ans: x = 1, 2/3

11. Simplify $\log_{10}((\sqrt[3]{5})^{\sqrt{18}} (\sqrt{20})^{\sqrt{8}})$ Ans: $2\sqrt{2}$